~~P~~ower conjugacy in Hig~~m~~an-Thompson gps

0. Story
1. Defining Thompson's gps
2. Alternative def: univ. alg
~~3.~~ 4. Conjugacy problem
~~4. Demo?~~

3. Orbit structure

Joint with Andrew Duncan & Nathan Barker
arXiv: 1503.01032
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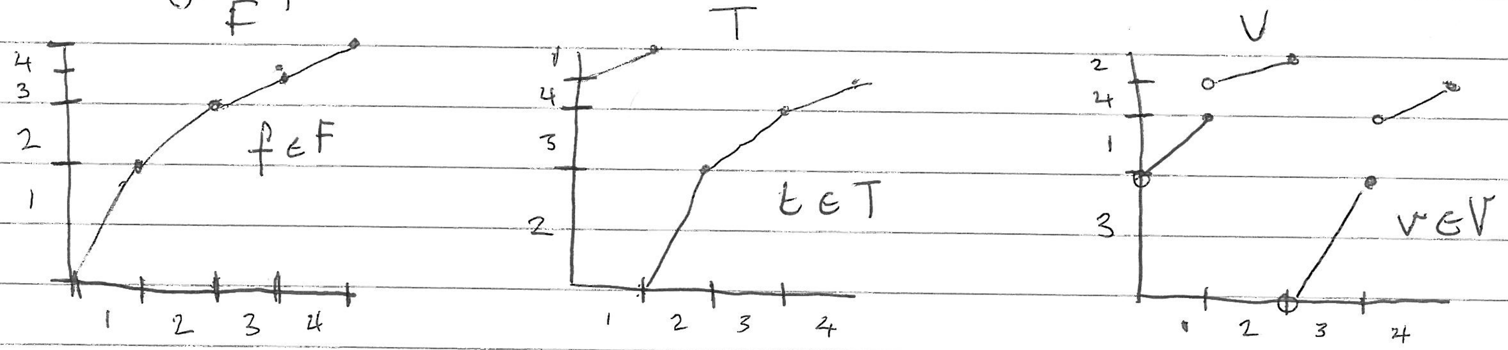
0. Story Nathan: PhD @ NCL working on Thompson's gps
Andrew: my supervisor & Nathan's internal examiner.
me → implement Nathan's pconj alg on computer
→ 3 months; gap in Higman's sol^n.

1. Thompson's gps $F < T < V$
All gps of f^{ns} $[0,1] \to [0,1]$ under composition.



"dyadic partition": repeatedly ~~halve~~ bisect interval (same # of chops)

- cts bijections $[0,1] \to [0,1]$
- increasing
- linear (affine) everywhere, except @ f'tly many breakpoints
- bkpts of $a/2^n$ $a, n \in \mathbb{N}$
- linear sections: gradient 2^m $m \in \mathbb{Z}$

- cts bijection $S^1 \to S^1$

right-cts $S^1 \to S^1$
(dis cts @ breakpoints)

F fixed order
T torus
V votever you vant

torsion-free torsion

So what?

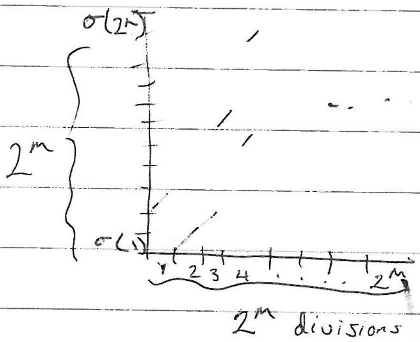
- Infinite
- Finitely Pres^{d}
- T, V simple (1st exx known)

  [Simple: if $N \trianglelefteq G$ then $N = 1$ or $N = G$.]

  "no interesting quotients"

- V contains a copy of every finite group

  $\underset{\text{Cayley}}{G \hookrightarrow S_n} \hookrightarrow S_{2^m} \ni \sigma$ perm



- Interesting (counter)-exx
- Resistant to attack.
- (Connected to all sorts of stuff)

More intro: James Belk's thesis (2007); Cannon, Floyd, Parry (1996)

2. ~~Typical~~ Alt def: univ. algebra

- Typical way to compute w/ elements is via a 'tree pair'
- Higman [74] advocated an algebraic approach.

$\mathfrak{U}$: a set cting an named elmt "x" "generator" $\longleftrightarrow$ $[0,1]$

closed under

unary op^{s} α_1, α_2 $\longleftrightarrow$ left/right subinterval

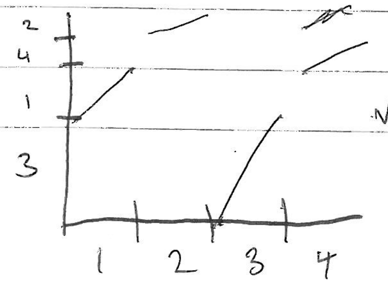
binary op λ $\longleftrightarrow$ "ordered union"

subject to laws $u_1 u_2 \lambda \alpha_i = u_i \quad i = 1, 2$

$(u\alpha_1)(u\alpha_2)\lambda = u$

($\mathfrak{U}$ is the "freest such thing" gen^{d} by x)

Specify elmt^{s} of ~~V~~(F, T) V ~~as~~ in terms of this alg



			φ		
				$\tfrac{1}{2}, \tfrac{3}{4}$	$x\alpha_1 \longmapsto$
$[0$	$\tfrac{1}{4}]$	$x\alpha_1\alpha_1$	$\longmapsto$	$x\alpha_2\alpha_1$	$(x\alpha_2\alpha_1)(x\alpha_2\alpha_2\alpha_2)\lambda$
				$[\tfrac{7}{8}, 1]$	
$\tfrac{1}{4}$	$\tfrac{1}{2}$	$x\alpha_1\alpha_2$	$\longmapsto$	$x\alpha_2\alpha_2\alpha_2$	$[\tfrac{1}{2}, \tfrac{3}{4}] \cup [\tfrac{7}{8}, 1]$
$\tfrac{1}{2}$	$\tfrac{3}{4}$	$x\alpha_2\alpha_1$	$\longmapsto$	$x\alpha_1$ $[0, \tfrac{1}{2}]$	
$\tfrac{3}{4}$	1	$x\alpha_2\alpha_2$	$\longmapsto$	$x\alpha_2\alpha_2\alpha_1$ $[\tfrac{3}{4}, \tfrac{7}{8}]$	

Thm $V \cong \mathrm{Aut}(\mathfrak{U})$
$v \longleftrightarrow \varphi$

φ called an auto^m b/c
- extends to a bijection $\varphi' : \mathfrak{U} \xrightarrow{\sim} \mathfrak{U}$
- homo^m: $\varphi(u)\alpha_1 = \varphi(u\alpha_1)$
  $\varphi(u_1)\varphi(u_2)\lambda = \varphi(u_1 u_2 \lambda)$

Why bother?
- rewriting strings & solving eq^{ns} instead of traversing trees
- much easier to concretely describe "orbit structure"

3. Conjugacy: Higman

Def $u \in \mathfrak{U}$, $\varphi \in V$ φ-orbit of u = dbl inf seq^n
$\dots \varphi^{-1}(u), u, \varphi(u), \varphi^2(u), \dots$

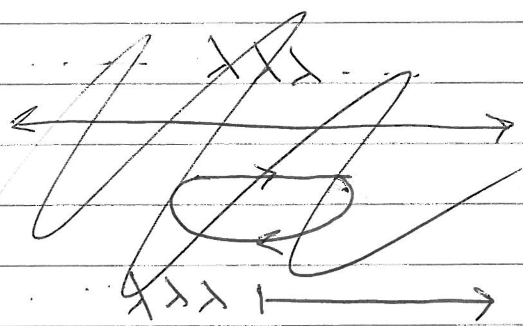
Ex $\varphi = f$ from before

$\dots \to [1/2, 1] \to [3/4, 1] \to [7/8, 1] \to \dots [1 - 1/2^n, 1] \to \dots$
$\dots \lambda \quad x\alpha_2 \quad x\alpha_2\alpha_2 \quad x\alpha_2\alpha_2\alpha_2 \quad x\underbrace{\alpha_2 \dots \alpha_2}_{n}$

$\boxed{\text{want to avoid } \lambda}$

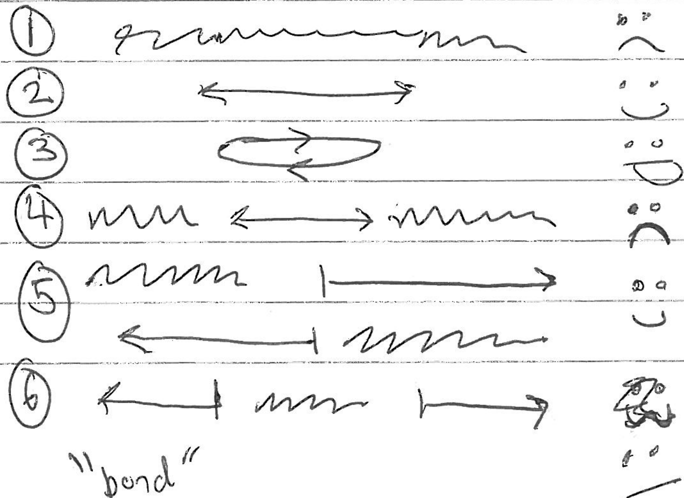
u_0 - something not involving λ

6 types of orbits: roughly speaking
~~incomplete~~



$\dots \lambda\ \lambda\ \lambda \dots$
$u_1\ u_0\ u_1 \dots$
$\dots u_1 \dots u_n\ u_0 \dots$
$?? \ \lambda\ u_1 \dots u_n\ \lambda\ ?? \dots$
$\dots \lambda\lambda\lambda\ u_1\ u_2 \dots u_n$
or $\dots u_{-n} \dots u_{-1}\ \lambda\lambda \dots$

$\dots v_{-n} \dots v_{-1}\ \lambda\lambda\lambda\ u_1 \dots u_n \dots$



"pond"

Higman ~~said~~ showed ~~that~~ we don't have to worry about

(1)

(4)

He said we don't have to worry about (6) (pond) but that's not true.

Higman's alg for conjugacy needs to be able to tell if two given elmts $u, v \in \mathfrak{U}$ belong to the same φ-orbit

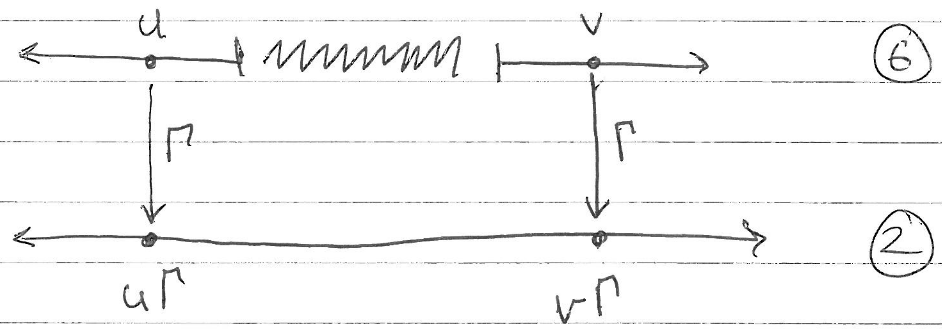
$\exists k \in \mathbb{Z}$ st $u = \varphi^k(v)$?

He did not explain how to do this for pond-orbits.

PATCH Pond orbit is always a finite distance above a type (2) orbit

Lemma

[...]



We test $\exists k \in \mathbb{Z}$ st $u\Gamma = \varphi^k(v\Gamma)$?

if $\nexists k$: u, v ~~do not~~ ~~share~~ do not share an φ-orbit

if $\exists k$: compute $\varphi^k(v)$ and see if it equals u.

if not, no other k will work.

☺

4 Higman's conj. alg. Each orbit is described by an eq^n.

Ex pdc : $u = \varphi^k(u)$

rsi : $u\alpha_2 = \varphi(u)$ } preserved by conjugation.

If $\psi = \rho\varphi\rho^{-1}$ i.e. ~~$\rho\psi = \varphi\rho$ then~~ $\psi\rho = \rho\varphi$

$\rho(u)\alpha_2 = \rho\varphi(u)$

$= \psi(\rho(u))$

$\bar{u} = \rho(u)$

$\bar{u}\alpha_2 = \psi(\bar{u})$

Thm Higman. A conjugator ρ st $\psi = \rho\varphi\rho^{-1}$ exists iff
$\exists$ a bijection b/w ψ-orbits & φ-orbits preserving

defining eqn
- type of orbit & associated data $[(\cdot, \alpha_2)]$
- relations b/w φ-orbits

Moreover, we need consider only a fte # of maps b/w orbits to see if such a bijection exists.

From here: brute force: — try every such bijection to see if any of them work.